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## Overview

### Overview for the Patterns unit of IMP Year 1 (2nd edition)

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Pacing Guides

Pacing guides for the Patterns unit of IMP Year 1 (2nd edition)

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## Patterns Calculator Guide

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[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## The Importance of Patterns

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## What's Next?

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## Past Experiences

### Intent

This individual activity is best used as homework, the first assignment of the school year. Including a writing assignment like this one will establish several expectations for the course.

- Purposeful homework will be assigned every day, and students' work on these assignments will be an important part of the course.
- Students will be asked to put their thinking—about mathematics and about themselves as learners of mathematics—to paper.
- All students' thoughts and ideas about the mathematics they are learning are crucial to the success of the course.
- Successful collaboration to do and learn mathematics is a key feature of this course.

### Mathematics

At first glance, this assignment does not look particularly mathematical. However, a growing body of research suggests that successful mathematical problem solvers are reflective thinkers. They know mathematics, and they know about mathematics as a discipline. They are aware of themselves as mathematics learners, and they can think about their own thinking—monitoring progress, evaluating strategies, choosing among skills and tools—while doing mathematics. Psychologists call this **metacognition**, and it is a hallmark of the thinking of effective problem solvers. In this activity, students are asked—perhaps for the first time (and certainly not the last time in this program)—to reflect on some of their experiences as mathematics students.

### Progression

This activity is designed to be done as homework after the first class and to be discussed, in small groups and as a whole group, in the next class.

## **Approximate Time**

10 minutes for introduction

20 minutes for activity (at home or in class)

10 minutes for discussion

## **Classroom Organization**

Whole class, then individuals, followed by small groups

## **Doing the Activity**

Take the time to share your expectations for this assignment and homework in general, including what you expect from students and what students can do if they don't understand an assignment. Telling students that you want to learn more about them and their backgrounds, and that you will not be grading their essays, but just recording whether they completed the assignment, may encourage them to do the assignment and to share honestly. One important goal of the first few homework assignments is to help students establish a pattern of doing their homework regularly.

Also impress upon students that they need to save their work throughout the unit, as they will be asked to include their written work on this assignment and others in the portfolios they will create at the end of this unit.

For the next day's discussion, you might want students to share their essays in their groups. If you plan to follow this suggestion, let students know now that other students will be reading their written work.

## **Discussing and Debriefing the Activity**

Students can read the essays of the other members of their groups. You might suggest that after reading each other's thoughts and experiences, students answer the Key Questions listed below, perhaps displaying these or

similar discussion questions on a transparency. Then students can share with the class the themes their groups encountered.

This is a good opportunity to reiterate that class participation—written, oral, and physical; in groups, individually, and with the whole class—is essential for success.

## **Key Questions**

**What are some of the important mathematical ideas you have studied?**

**How are your group's ideas about your most and least helpful learning experiences similar? How are they different?**

**How are your experiences, thoughts, and feelings about working with others similar? How are they different?**

## POW 1: The Broken Eggs

### Intent

As the first POW, or Problem of the Week, **The Broken Eggs** is students' first opportunity to work on a substantial problem over several days and communicate the results of their work in writing, using a format that will carry across the four years of the program. (See "Problems of the Week" in the Overview to the Interactive Mathematics Program.) [Link to the "Problems of the Week" portion of the Overview.]

### Mathematics

This POW is a version of a well-known problem in number theory. Here is a translation from a seventh-century text written by the Hindu mathematician Brahmagupta: **An old woman goes to market, and a horse steps on her basket and crushes the eggs. The rider offers to pay for the damages and asks her how many eggs she had brought. She does not remember the exact number, but when she had taken them out two at a time, there was one egg left. The same happened when she picked them out three, four, five, and six at a time, but when she took them out seven at a time they came out even. What is the smallest number of eggs she could have had?** A similar problem was posed by the Chinese scholar Sun Tsu Suan-Ching in the third century: **There are certain things whose number is unknown. Repeatedly divided by 3, the remainder is 2; by 5 the remainder is 3; and by 7 the remainder is 2. What will be the number of things?**

In the activity, students search for numbers divisible by 7, but when divided by each of numbers 2 through 6 leave a remainder of 1. To find solutions to this problem, students must examine multiples of 7 and remainders when dividing by 2 through 6, and reason about patterns in these results. **The Broken Eggs** problem has many solutions, creating a complex task that will allow any high school student to begin to work on the question and all to pursue it as far as their interest (and time) allows (see "**About Solutions to Activities**" [link to About Solutions to Activities in the Overview] in the Overview to the Interactive Mathematics Program).

## **Progression**

Students will work on this POW primarily outside of class. This unit is carefully designed to support student success, especially with this first long-term, problem-solving and writing project. The problem is posed early in **The Importance of Patterns** and revisited at several points over the next few class meetings. Three students will present their solutions to the class, and all will turn in their written work.

## **Approximate Time**

5 minutes for introduction

30 minutes for groups to begin exploration

20 minutes for introducing POW write-ups

30 minutes for individuals (homework; do and write “Process”)

5 minutes for small-group discussion

30 minutes for individuals (homework; complete “Process”)

15 minutes for discussion findings and of next phases of write-up

15 minutes for discussion of presentation expectations

30 minutes for individuals (homework; complete POW write-up)

15 minutes for presentations

## **Classroom Organization**

Individuals and small groups, concluding with whole-class presentations and class discussion

## **Materials**



Presenters will need presentation materials, such as transparencies and pens, a few days prior to the due date.

## Doing the Activity

Please be thoughtful about the extended timeframe over which the activity occurs. The more support you offer students in this, their first long-term and significant problem-solving and writing activity, the more successful they will be in the subsequent writing activities they will encounter.

The commentary below divvies the focus of this supportive work into several “phases.” This is not meant to indicate sequential days; depending on your scheduling situation, consider these the segments necessary to support student work, however you parse it into your daily plans.

## Introduce the Problem

You might begin this activity by having one or more volunteers read it aloud, through the section “Your Task.” Tell students that they will be working on this problem in groups, sharing ideas and insights. They should keep notes on how they and their groups work on the problem, as they will be discussing these things in their final write-ups.

## Begin Exploring

Have students work on the problem in groups. Although some students might find a solution during this initial work, this is not an expectation for the first day.

If groups are stuck, ask what they have tried. It is important that students have time to reach conclusions at their own pace. To clarify whether they understand the problem, you might ask why 49 is not a correct answer. You might also suggest they consider this simplified problem: **Suppose the farmer remembered that only when she put the eggs in groups of either two or five, there was one egg left over. What would be some possibilities for the number of eggs in that situation?**

If any groups find the answer 301 today, you can urge them to look for other solutions. If groups need further challenges, they can look for a general solution or a description of how to find other solutions, and then for an explanation of how they know their general solution includes all possibilities.

## **The Standard POW Write-Up**

Have students read The Standard POW Write-up. Most POWs will have a “Write-up” section that uses the basic components listed in The Standard POW Write-up. The write-up instructions will often simply refer to these components by name, giving additional details only when the write-up differs from the basic model provided here. You may want to post the five write-up components on the wall.

Have students work individually for a while to create problem statements for the POW. Then have them share their ideas in their groups, and have each group use these ideas to create the best problem statement possible.

Ask one or two groups to share their problem statements with the class. In the discussion, bring out that the problem statement should not simply repeat the problem as originally stated, but should try to focus on the essentials. You might work with students to distinguish between the “story” aspects of the problem and its mathematical core. With this first POW, students might include both aspects in their problem statements, but over the course of the curriculum, they should gradually move toward an emphasis on the mathematical elements of the POWs.

You might remind students that taking notes as they work will help with the “Process” part of their write-ups. Encourage them to collaborate with classmates. You might mention explicitly that you do not consider it cheating to work with someone else on a homework assignment or POW, as long as students acknowledge that collaboration in their write-ups. On the other hand, students should not simply copy each other’s work or allow others to copy from theirs. You might also offer advice about how to help each other, such as by giving a hint or asking a leading question. Point out

that if they give a friend the answer, they deprive the friend of much of the learning experience.

## **Begin Writing the “Process”**

Two or three days after assigning this first POW, assign for homework simply working on the POW for 20 to 30 minutes. Tell students they should come to class tomorrow with a portion of their “Process” written. If they are using word-processing software for their write-ups, they should bring a printout of this draft.

Encourage students to keep notes about ideas they have and things they try and to pause occasionally to add to the “Process” portion of their write-ups. A structure such as work 7 minutes, write for 3 minutes might help students achieve this goal.

When students return to class, encourage them to share ideas in their groups. Then explicitly instruct them to share what they have done in writing the “Process.” It can be valuable to have students pass their write-ups around their groups to see how others are recording their solution methods.

## **Finish Writing the “Process”**

On the very next day (the day the homework above is due), have students spend 20 to 30 minutes working on the problem and completing the write-up of the “Process.” Again, they should bring a draft to class.

In class, again encourage groups to share findings from their investigations the previous evening, including reading one another’s “Process,” perhaps in pairs this time.

Follow this with a class discussion. Ask students to describe what they think is in the “Process” section, as they have written it over the past few days. Return to the **The Standard POW Write-Up** reference page, and ask students to compare their impressions with the description of the “Process” section here.

Remind students that they have three sections left to write: “Solution,” “Extensions,” and “Self-assessment.”

## Completion and Presentation Preparation

Use the contexts of **Marcella’s Bagels** and **Extended Bagels** to remind students of the write-up structures and expectations.

As this will be the first POW presentation, ask for three volunteers to present their work. You may find it easier to get volunteers if you mention that this first group of presenters will get some extra guidance. (For future POWs, either select students at random, choosing from among those who have not yet done POW presentations, until everyone has had a turn, or ask for volunteers, again explaining the expectation that everyone will present once before cycling through again.)

Discuss with the class what will be expected of presenters and of the audience. Emphasize that presentations are to be discussions about **ideas**. It is important that presenters prepare to share what they learned about the problem and not feel pressured to present “the” answer.

Audience members should listen to discover what presenters have figured out, how they approached the problem, and the reasoning behind their conclusions. The audience will be expected to ask clarifying questions, such as “I don’t understand how you arrived at this conclusion; I seem to get \_\_\_\_” or “That idea seems to contradict \_\_\_\_.”

Presenters are to use transparencies to **help with** their presentations, rather than to **be** their presentations. In other words, they will not present only what is on the transparencies, but should plan to explain the problem, using the transparencies to save the trouble of writing as they talk. They can include diagrams, numeric calculations, and whatever else might be helpful.

Also, presenters may need reminding to plan to talk about **all** parts of the write-up, not only about their solutions. (In later POW presentations, students may find certain sections need less attention, especially given only

5 minutes to present.) Finally, encourage them to make any writing on transparencies large enough to be readable.

You might meet briefly with volunteers to address any questions or concerns they have and to give them transparencies and pens for preparing their presentations.

Once presenters are selected and expectations for the presentations have been communicated, tell everyone that the homework is to finish the write-up.

## **Discussing and Debriefing the Activity**

### **Presentations**

Because making presentations is very difficult for some students, these first POW presenters might have a tough time. Because of their willingness to volunteer, they deserve special consideration from the audience and assistance from the teacher.

Briefly remind the audience of these expectations (which you may want to post):

- Acknowledge the effort and courage of the presenter, regardless of the quality of the presentation.
- Treat each other with respect and listen attentively.
- Listen for what the presenter learned, ask questions when you don't understand, and challenge things you think are incorrect. Being respectful does not mean being passive. It is not disrespectful to question, add to, or challenge each other's work if it is done in the proper spirit.

**Multiple routes to the solution:** Encourage students to ask questions during the presentations. After all the presentations are over, ask if anyone has anything else to add. Be sure students realize that this invitation includes presenting a different method for finding or explaining an answer—they do not have to have a new or a different answer.

**More than one solution:** If the presentations did not deal with the issue of the POW having more than one answer, bring that up now. Many students may have stopped exploring when they found that 301 fits all the given conditions. An important mathematical question to ask is, **Is this problem one of those that has more than one answer?** This problem, like many others, does have more than one answer.

It is not necessary at this time that students find the general expression for all possible solutions, but they should recognize the possibility of multiple solutions. The supplemental activity **More Broken Eggs** asks students for the general solution.

## Key Questions

**How do you know that (301, for example) is a solution? Is not a solution?**

**Do you suspect there are other solutions? Why?**

## Supplemental Activity

**More Broken Eggs** (extension) expands on **The Broken Eggs**, in which students found a possible number of eggs the farmer might have had when her cart turned over. The task now is to look for other solutions, to find and describe a pattern for obtaining all the solutions, and to explain why all the solutions fit that pattern.

## Samples of Student Work

Sample 1: Page 1 [[Link to Broken Eggs Student 1a](#)] Page 2 [[Link to Broken Eggs Student 2a](#)]

Sample 2: Page 1 [[Link to Broken Eggs Student 2a](#)] Page 2 [[Link to Broken Eggs Student 2b](#)] Page 3 [[Link to Broken Eggs Student 2c](#)]

Sample 3: Page 1 [[Link to Broken Eggs Student 3a](#)] Page 2 [[Link to Broken Eggs Student 3b](#)]



## Who's Who

### **Intent**

This activity is included early in the unit to engage students in the important processes of logical reasoning and proof.

### **Mathematics**

This activity presents interlocking sets of conditions. Using these conditions, students must identify “who’s who” and are asked to provide a convincing argument—a proof—of their conclusion. Issues of proof arise repeatedly throughout the curriculum and in daily interactions. Most significantly, students are always expected to justify their solutions, to convince others, and to be convinced by others.

The reasoning students will use to analyze the stated conditions, to make conjectures about the solution, to test those conjectures to convince themselves that their solution meets the stated conditions, and then to determine whether their solution is unique—that is, to prove their solution—is at the heart of what it means to do mathematics.

### **Progression**

Students are asked to find a solution to this puzzle and to determine whether that solution is unique. The activity also gives students the chance to use two components of POW write-ups: “Process” and “Solution.”

### **Approximate Time**

5 minutes for introduction

20 minutes for activity (at home or in class)

15 minutes for discussion



## **Classroom Organization**

Individuals, followed by whole-class discussion

## **Doing the Activity**

Review what is expected in the activity. Urge students to start taking notes as soon as they begin thinking about the problem and to use those notes in the “Process” portion of their write-ups. Emphasize that the “Solution” part of their write-ups must demonstrate how they are certain of their solution and especially how they know that it is the only solution. (Some students might be unfamiliar with the game of Hearts, but they don’t have to know anything about cards or this card game to do this activity.)

## **Discussing and Debriefing the Activity**

Some students might approach this activity using logic. For example, because Felicia passes her cards to the ninth grader, she can’t be the person who passes to the eleventh grader.

Others might list all possible cases. With only three students, it is not too difficult to list all possible grade levels and all possible arrangements of the students around the table, and then see which one meets all the conditions.

Whatever approach they use, students have the opportunity to engage in clear, logical argument to explain why their solution is unique.

Give students a few minutes in their groups to compare how they solved Question 1. Suggest that they focus not only on the answer, but also on how they worked on the problem—that is, on the “Process” component of the write-up.

As you circulate and listen in on the discussions, identify students who approached various parts of the activity in interesting ways, and ask them to present those parts to the class. Try to get several methods presented, both to describe the approaches themselves and to emphasize that there are many possibilities.

When students are convinced that there is a single solution, raise the issue of whether there are other answers, and ask students to explain how they can be sure there is only one. **Is this one of those problems that has more than one answer?** As with the discussion of solution methods, encourage different approaches.

After several students have offered explanations of why the answer is unique, ask whether students are completely convinced by these arguments. **How convinced are you?** Use this opportunity to review the word [proof](#). Clarify that in this problem, a complete proof involves two aspects:

- Showing that the particular seating arrangement satisfies the conditions in the problem; that is, that a solution **exists**
- Showing that no other seating arrangement fits those conditions; that is, that the solution is **unique**

## Key Questions

**Is this one of those problems that has more than one answer?**

**How convinced are you?**

## Supplemental Activities

**The Number Magician** (reinforcement) asks students to determine the original number that produces one particular answer and to analyze the method used to determine the original number so quickly.

**Whose Dog Is That?** (extension) is a logic puzzle much like **Who's Who?** Students are given several interlocking conditions and must use logical reasoning to determine a set of conclusions.

## The Standard POW Write-Up

### **Intent**

This reference page introduces students to Problems of the Week and to the standard POW write-up. Students will also refer to this reference page throughout the year to aid with their POW write-ups.

### **Mathematics**

Communicating about mathematical thinking is an important part of doing mathematics. This reference page is designed to support students' written communication about their findings when exploring large mathematical problems. Stating the problem, discussing one's methods, and concluding succinctly and with justification, so that a reader will understand what has been written, should be the goal of every student writer. By suggesting extensions to the problem, students will be saying that the mathematics has not been fully explored, given the time constraints. Their self-assessment will be an evaluation of the effort and quality of their work, what they take pride in, and what they wish they could have done better.

### **Progression**

Initially, this reference page will support students as they assemble a paper that communicates the work they did and what they learned. As they become more comfortable with the process of writing about mathematics, the page should be returned to and discussed occasionally. It will prompt students to think more deeply and to share ideas with other students about what each section of the POW write-up means, to evaluate each others' papers, and to focus on improving portions of their own work.

### **Approximate Time**

20 minutes for discussion

## Inside Out

### Intent

This activity introduces students to a powerful representation of functions, In-Out tables, that will be used repeatedly throughout the IMP curriculum. The In-Out machine metaphor is used to introduce In-Out tables and the terms **input** and **output**. In their exploration, students experiment, make conjectures, and work toward getting clear verbal statements of the rules.

### Mathematics

A powerful way to think about a [function](#) is as a machine. The thing put in is called the [input](#) (the **In**, for short); the thing that comes out is called the [output](#) (the **Out**). An [In-Out table](#) is one **representation** of a function. Other representations include graphs and symbolic rules. (Students will study these representations in **The Overland Trail**.) We often use the word **function** in such phrases as “the **Out** is a function of the **In**,” which means that the **Out** value depends on, or is determined by, the **In** value.

### Progression

This activity works well as homework. Students are introduced to the idea of an In-Out machine and learn to keep track of what it can do by using an In-Out table. They attempt to decode some In-Out tables and then create two of their own.

### Approximate Time

15 minutes for introduction

20 minutes for activity (at home or in class)

45 minutes for discussion

## Classroom Organization

Whole-class introduction, followed by individuals, then small groups, concluding with whole-class discussion

## Doing the Activity

Introduce the idea of an **In-Out machine**. Many students may have been exposed to this method of representing functions, but others may not have been. Explain that something, often a number, is put into the machine. The machine does something to that object, and something comes out. The thing put in is called the **input** (the **In**, for short); the thing that comes out is the **output** (the **Out**). With your first example or two, you might show inputs as if they were actually being put into a machine and outputs as if they were coming out of the machine.

A nice way to start is to have students try to figure out what the machine is doing without any initial information. For example, you can ask, **What happens if I put in the number 5?** Have students make guesses, though they will probably recognize that they can't possibly know for sure. Offer clues, such as "too high" or "too low," until someone gets the answer you have in mind. You may want to have several options in mind at first so that it takes more than one guess.

Then ask for another number to use as the input and have students guess the corresponding output. Continue in this way, as students gradually get additional information about your mystery machine. At some point, ask, **How might you keep a record of the information?**

When most students seem to have figured out the rule, demonstrate how to organize the information using an In-Out table. For example, you might have a table like this for an "add 4 machine."

In	Out
5	9
3	7
0	4
9	?

**Can you figure out a rule for this table?** Ask students to tell you a rule in words. Work toward getting them to express rules completely, such as “The **Out** is 4 more than the **In**” or “You get the **Out** by adding 4 to the **In**” rather than just “Add 4.” Write the rule next to the table.

Offer another example of an In-Out machine, and develop another table of student guesses. Name the input and the output parts of the table, using the metaphor of the machine, and name the table an In-Out table.

Next, give students an In-Out table, and ask them to guess the rule for the In-Out machine that the table is associated with. Again, write the rule as, “The **Out** is . . .”

## Discussing and Debriefing the Activity

Ask students, in their groups, to share the rules they found for the In-Out tables in Questions 1 to 5. Also ask each group to prepare a transparency of one of the tables. Explain to the class that as they prepare to present, you would like them to focus more on their thinking process for identifying the rule than on the missing numbers. Also, presenters should write each rule as a complete sentence, beginning with “The **Out** is . . .”

During the presentations, encourage the audience to ask how the presenting group found each particular missing item or rule.

**Questions 1 and 2:** These two numeric tables represent what students will later classify as linear functions. Some students may have trouble finding

the pattern in Question 2. Ask students who have found it to explain, if they can, the process they used to discover a rule to fit the information.

**Question 3:** The rule generally used for this table is that the **Out** is 1 less than the number of letters in the **In**. Based on this rule, you may want to bring out that there are many possible choices for the missing inputs, but only one choice for each missing output.

**Question 4:** This is probably the most challenging In-Out table in the activity, as it does not fit any standard idea of what constitutes mathematics, and because there is no simple algorithm for finding a relationship between the inputs and the outputs. As there is no explicit numeric information in the inputs, the first stage in thinking about this problem is to identify something in the pictures that can be associated with numbers. Students may have a variety of ideas about how to do this and will then need to find a rule that connects the numeric information in the pictures to the numbers as outputs.

If everyone is stuck on this problem, ask what changes from picture to picture and use this information to build a new table. For example, students might focus on the number of eyes, in which case they can see the table in Question 4 as equivalent to this table.

In	Out
1	3
3	11
4	15
?	7

**Question 5:** One rule that works is that the **Out** is the second vowel of the **In**. But students may have other ideas. Some may have decided, based on the first three rows, that the **Out** is the first vowel of the **In**. If so, use this opportunity to remind students to check their rules against all the given information.

Another possibility that may arise is that the **Out** alternates between the fourth and third letter of the **In**. That is, I is the fourth letter of **division**, E is the third letter of **ever**, O is the fourth letter of **opportunity**, and so on. If this or a similar suggestion arises, bring out that in this pattern, the **Out** depends on the sequence in which the input values occur rather than only on the value of the **In**. If the order of pairs shown in the table is changed, this pattern will no longer exist.

Make sure to discuss the “can’t be done” entry for Question 5. Ask students what sense they made of it. They might respond with statements such as these.

- To get “can’t be done” as the **Out**, you need to put in a word with only one vowel.
- You can only use words with at least two vowels for this table, so words with only one vowel can’t be done.

Before introducing the term [function](#), it is important to identify the distinction between functions and arbitrary tables of data. To illustrate, ask students what they think about a table like this one.

In	Out
3	5
2	8



Bring out that there is something unusual here, as there are two different outputs for the same input. In terms of the metaphor of an In-Out machine, you might identify this as a “broken machine.”

With this as background, introduce function as the formal mathematical term that roughly corresponds to the idea of a “working” In-Out table. You might also use the phrase function machine as another term for an In-Out machine. The key idea is that a function must be consistent. That is, it must give the same output every time a particular input is used.

A related idea is that the output should not depend on where a given input is listed in a table. So a rule such as “The Out alternates between the fourth and third letter of the In” (see the earlier discussion of Question 5) does not describe a function.

Also include a case in which different inputs have the same output, such as Question 5. In other words, bring out that functions can’t produce different outputs for the same input, but they are allowed to produce the same outputs from different inputs.

Explain that the concept of a function is one of the major unifying ideas of mathematics and that students will be working with functions throughout their mathematics program. You might mention that rule, table, and function are often used almost interchangeably in informal mathematical work, even though the terms technically have different meanings.

Introduce the term [domain](#) for the set of things that are allowable as inputs for a given In-Out table. Ask, **What things are allowable as inputs for each table in last night’s homework?** You can bring out that in Questions 1 and 2, the In must be a number, while in Questions 3 and 5, it must be a word (or perhaps any sequence of letters). In Question 4, the **In** should probably be a picture similar to those shown.

In their work with In-Out tables, students have used such rules as “The **Out** is twice the **In**” or “You get the **Out** by adding 5 to the **In**.” In the context

of a specific example, you can bring out that a table will generally show only an incomplete picture of a function. It may have enough information to strongly suggest how the rule works, though students will already have seen that this is open to interpretation. But even if we settle on one specific rule, we can't tell from the table exactly what the domain is. Again, we can make an assumption about this, but usually it is only a guess.

Most often, the domain is an infinite set, and thus the function consists of an infinite number of In-Out pairs. Students should recognize that the table can only display a few of these pairs. You might indicate that because of this, we say that the table represents the function, but that technically the function is more than what is shown in the table.

Introduce the term **range** for the set of things that can be outputs for a given In-Out table. This set depends on what the domain is. For instance, for the doubling rule, if the domain is restricted to the whole numbers, then the range consists of the even whole numbers; but if the domain also includes positive fractions, then the range includes all whole numbers as well as all positive fractions.

**Question 6:** Have students exchange their In-Out tables and look for rules for the tables their fellow group members created. Each group can copy onto a sheet of poster paper two or three favorites from among the tables they created. They should make their tables big enough so that the entire class can read them when the poster is on the wall.

When groups have displayed their posters, they should attempt to find rules for the tables posted by other groups.

Ask the class whether there are specific examples they want to discuss or with which they had difficulty. You can have the group that created the problem or students from other groups offer hints on how to find a rule.

## Key Questions

**Did you see a method for finding the missing input?**

**What makes this problem difficult?**

**What do you think about this table?**

## **Supplemental Activity**

**A Fractional Life** (reinforcement) is part of **The Greek Anthology**, a group of problems collected by ancient Greek mathematicians.

## Calculator Exploration

### Intent

Graphing calculators will be part of students' tools for doing mathematics throughout the IMP program. This activity will give some students their first chance to learn how these calculators work. It also offers students an opportunity to make a short presentation of something they have discovered.

### Mathematics

Given the versatility and power of the graphing calculator, and the wide variety of prior experiences students are likely to bring to this open-ended activity, students' explorations will probably range widely. However, there are some important mathematical issues students will encounter.

- How to handle order of operations on a calculator
- Calculators will evaluate such expressions as  $17 - 6 \div 3 + 4 \times 9^2$ , without the need for parentheses, by doing the exponent first, then multiplication and division, and finally addition and subtraction.
- How to use the built-in mathematical functions
- To find the square root of 3 you must access the square root function before the number 3, but to find 5! you must access the factorial function after the number 5.
- $\sin(30)$  will not be 0.5 unless the mode is set to degrees.
- The graphing capabilities of the calculator.

### Progression

Students will explore their calculators in pairs and then share discoveries with the class.

### Approximate Time

20 minutes for activity

20 minutes for discussion

## **Classroom Organization**

Pairs, followed by whole-class discussion

## **Materials**

Calculator guidebooks or manuals

TI Calculator Basics (optional) [[download PDF](#)]

Overhead calculator

## **Doing the Activity**

Tell students that they will be using a graphing calculator or handheld very often in their math class—so often that it will become a tool with which they think and explore, rather than simply calculate.

Have students read the activity on their own, and then highlight that they are to work with a partner to learn whatever they can about the calculator and, later, to demonstrate something they learned.

Give students time to work on this open-ended activity on their own, free from intervention, for a while. Through this experience, they may come to understand that they can learn about calculators, by trial and error, which will help them feel confident with these tools in the future.

Students should focus on simply learning how their calculator works. If you notice a pair fretting about not learning something in particular, encourage them with a reminder that the goal is to explore the calculator until they discover something new. You might also suggest that they explore a button that looks interesting to them. Or ask them to think of something they frequently do in math class and see if they can figure out how to do it on this calculator.

Encourage students to be thoughtful in their preparations to present. Assure them that their presentations can be simple, such as, “If you press this key, such-and-such happens.” They don’t necessarily have to learn how to accomplish something useful to make the information worth reporting.

Students may want to use manuals to learn how to do a specific activity or to find out what kinds of things the calculator can do. As you circulate, suggest to individual students that they prepare presentations on particular topics.

## **Discussing and Debriefing the Activity**

Give pairs a short time to organize what they learned and to prepare their presentations. Pairs should probably be ready with several ideas to present, so they will have something available if another group presents one of their ideas.

Have pairs make their presentations in an appropriate sequence. For instance, schedule presentations that focus on more elementary aspects of calculator use before those on more advanced or obscure topics.

Students should present using an overhead calculator or appropriate software.

## Pulling Out Rules

### Intent

This activity gives students more opportunities to find and express rules for In-Out tables, both in words and symbolically, and to use an In-Out table as a problem-solving tool.

### Mathematics

- This activity raises the several mathematical issues.
- Finding rules for In-Out tables.
- Developing symbol sense. For example, “The **Out** is 2 times the **In**, then add 3,”  $\text{Out} = 2 \text{ In} + 3$ ,  $\text{Out} = 2x + 3$ , and  $y = 2x + 3$  are equivalent and increasingly abstract ways to use symbols to summarize the rule for Question 1a.
- Confronting the idea that the number of data points in a table affects the number of rules that will explain all the data.
- Using a function as a mathematical model of a quantitative situation, and then using the model to solve a problem related to that situation.
- Introducing the terms variable, algebraic expression, coefficient, and constant term.

### Progression

Students are first asked to find rules for In-Out tables that each contain four pairs of numbers. Then they are asked to generate many possible rules that fit tables with only one or two pairs of numbers. Finally, they are presented with a problem for which an In-Out table is a helpful solution tool.

### Approximate Time

25 minutes for activity

20 minutes for discussion

## **Classroom Organization**

Individuals, then groups, followed by whole-class discussion

## **Doing the Activity**

Tell students that in this activity they will look for rules for more In-Out tables.

## **Discussing and Debriefing the Activity**

Students should begin by asking questions of and comparing results with their group members. During this time, ask for some groups to prepare presentations for one of the tables.

During presentations, encourage students to talk about the strategies they used to find their rules. Ensure that rules are presented as complete sentences, and record these sentences on the board.

Now that students have worked with a variety of In-Out tables, it may be valuable to let them share techniques that they have found for finding rules. As they do, help them to clarify their own thinking and encourage them to challenge each other to explain their ideas clearly.

Make sure a variety of rules for each table are shared. In Question 2, with only one data point to explain, students can find an infinite number of possible rules. If an In-Out table contains two points, as in Question 3, there are still infinitely many possible rules, but there is now only one linear rule that will work.

Question 4a asks students to create an In-Out table to analyze a real-life situation.

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Number of volunteers	Number of bags of weeds pulled
1	2
2	5
3	8

Some students will answer Question 4b by extending the table using the pattern “Add 3 bags for each additional person” until they reach an output that is large enough. Some might find and employ a rule that fits the table  $\text{Number of bags} = 3 \cdot (\text{Number of volunteers}) + 1$ .

Using either approach, students will discover that 30 does not appear as an **Out** if they stick to whole numbers for the **In**. **So what should the supervisor do?** To address this question, students must make sense of their work so far in light of the problem context. Their ideas could include these.

- The supervisor should “play it safe” by getting 11 volunteers, taking into account that the job might actually involve more than 30 bags of weeds and that some volunteers might work faster than others.
- The supervisor should get 10 volunteers and either make them work extra hard or join in the work as needed to finish the job.

There is no right answer; each of these suggestions (and other possible ideas) makes sense in this context.

Begin moving students from rules expressed in words to rules expressed symbolically. The discussion may be richer if this is a two-operation rule, such as “Get the Out by tripling the In and then adding 1.”

Have students find the Out that goes with each of several In values for their rules. For the rule just stated, the table might look something like this.

In	Out
1	4
3	10
6	19
5	16

Help students record the rules as [algebraic expressions](#). Students already know symbols for the numbers and operations involved, and most will have used symbols to replace unknown quantities. Remind them how much they know already about writing algebraic expressions.

With a few volunteered responses, students should arrive at an algebraic expression that all agree finishes the sentence. Record the expression in the In-Out table.

In	Out
1	4
3	10
6	19
5	16
t	$3t + 1$

Below are a few additional ideas about algebraic notation and terminology that you can either elicit from students or simply present, using the context of the table just discussed.

- It is conventional to abbreviate  $3 \times t + 1$  as  $3t + 1$ .
- It is acceptable to write  $t \times 3$ , unusual to write  $t 3$ , and most common to write  $3t$ . Emphasize that omitting the multiplication sign is simply a convention of notation—that is, an agreement among mathematicians to write things a certain way. There is nothing inherently wrong about using a multiplication sign between a number and a variable or about placing the variable in front of the coefficient—it's just not usually done that way.
- The letter  $t$  is a **variable**. Rather than formally defining the term, you might just say that a variable is a letter that is being used to represent a general case.
- $3t + 1$  is an **algebraic expression**. A number used to multiply a variable, such as 3 in the expression  $3t + 1$ , is a **coefficient**.
- A number by itself that is added or subtracted in such an expression, such as 1 in the expression  $3t + 1$ , is a **constant term**.

After introducing this terminology and notation, you might ask students to express some of the other rules they found for the tables in Questions 2 and 3 in algebraic form. Using a variety of letters for the inputs emphasizes that the particular letter chosen has no significance.

## Key Questions

**What rules have we found for this table? Are these rules really different? That is, would they lead to different tables? Or are they different ways of stating the same rule?**

**What should the supervisor do?**

## Lonesome Llama

### **Intent**

This activity is designed to promote student cooperation and communication about mathematics by making the process of working together on a mathematical task an explicit learning focus.

### **Mathematics**

Students will, within certain constraints, be trying to identify the unique card in a stack of 46 cards. The characteristics that distinguish the cards are mathematical—such as the number, type, and orientation of geometric figures—so students will be communicating about mathematics. The mathematical goals of this activity are for students to develop ways to describe the distinctive features within a set of diagrams that are largely alike and to develop a procedure for sorting the diagrams by those features.

### **Progression**

After students, working in small groups, have found the singleton card or made sufficient progress, a whole-class discussion can focus on how they worked and what they discovered.

### **Approximate Time**

30 minutes

### **Classroom Organization**

Groups, followed by whole-class discussion

### **Materials**

46 Lonesome Llama cards for each group [download PDFs]

## Doing the Activity

Understanding a bit about group dynamics can make a group a better team and enable students to get more out of the experience. The main purpose of **Lonesome Llama** is to get students to look at group processes and roles while they are engaged in problem solving. Everyone must participate in order to complete the task successfully.

Before passing out the sets of cards, have students read through the entire activity, and take some time to review the rules. Emphasize that students don't get to look at each other's cards until the activity is completed (that would make the activity way too easy!) and that what students say about how they work with each other is as important as what they learn about the cards.

Then hand out one set of cards, face down, to each group, and ask a student to deal out the cards approximately equally among the group members. (Because there are 46 cards per set, students in a given group won't all get the same number of cards.) Each student can then look at his or her own cards. Although students will have read the rules, you will likely need to review the rules one at a time, with students looking at their own cards, to ensure that everyone understands them.

Circulate as students work, ensuring that they follow the rules and attend to how they are working together.

You might instruct the members of groups that finish early to respond in writing, privately, to this prompt: **What makes a group work well?** When they finish writing, you might have them begin the activity **Role Reflections**.

## Discussing and Debriefing the Activity

Ask students for comments about the activity:

**How did you know when you were done? How confident were you in knowing you had solved the problem? Why were you so confident?**

**What mathematics involved in this activity? What else was mathematical about the ways your group worked?**

This last question can be an opportunity to mention that mathematics involves not only knowing terms and facts, being able to use them efficiently and accurately, and solving problems; but also being able to reason, communicate, and share ideas with others so that you can do things as a team.

Ask for volunteers to share their ideas about the prompt, **What makes a group work well?** Students may choose to read what they wrote or may prefer to talk about their thoughts.

You may want to work together to create a poster entitled “Characteristics of a Well-Functioning Group.” Such resources, which can be developed throughout the unit and the entire year, are useful for asking students to reflect on how their groups are working or to consider what role they can take to ensure that their group functions well. You might begin by asking students to privately list things they would see happening in a well-functioning group. Then have students volunteer ideas, while you record them on the board or chart paper, until their lists are depleted.

## **Key Questions**

**How did you know when you were done?**

**How confident were you in knowing you had solved the problem? Why were you so confident?**

**What mathematics was involved in this activity?**

**What else was mathematical about the ways your group worked?**

## **Supplemental Activity**

**Counting Llama Houses** (extension) asks students to identify the ways in which the houses in **Lonesome Llama** differed and then to determine how many different houses could have been created using these variations.

## Role Reflections

### Intent

This activity draws upon the **Lonesome Llama** experience in such a way that students recognize the various roles that members must assume in order for a group to function well.

### Mathematics

IMP activities are designed to encourage a high degree of interaction, involving collaborative problem solving as well as engaging discussions of ideas, arguments, and presentations. For such a learner-centered environment to be effective, care must be taken to help students work in groups effectively. **Role Reflections** helps students become aware of these norms, or roles, for working in groups and why they are important. See [“Assigning and Using Roles in Cooperative Groups”](#) in the Overview to the Interactive Mathematics Program for more information.

### Progression

Upon completion of **Lonesome Llama** (or as a filler while groups finish **Lonesome Llama**), students should privately reflect on the activity to identify actions taken by a student engaged in the various roles. A class discussion of responses helps to highlight the value of the variety of contributions that class members made to the successful completion of the activity.

### Approximate Time

15 minutes

### Classroom Organization

Individuals, then groups, followed by whole-class discussion



## Doing the Activity

When the students in a group have determined that they are confident they know the **Lonesome Llama**, provide the following instructions. Ask each group member to think back to the activity and remember the various actions taken by their group members that helped them complete the task. Instruct students to organize their recollections using the roles listed in **Role Reflections**. This work should be done individually and privately.

Once **every** member of a group has completed identifying **one** person for each role, have the group members share what they recorded and what moment during the **Lonesome Llama** activity prompted them to identify each person.

## Discussing and Debriefing the Activity

Once every group has spent at least some individual time identifying the moments that a member took on a particular role, bring the class together for a discussion. Ask for volunteers to share actions taken by fellow group members that moved them forward in their success to complete the **Lonesome Llama** activity. Ensure that both **task roles** and **socio-emotional roles** are shared.

## Key Questions

**Which role might someone in your group have performed that would have helped the group be successful with the activity?**

**Were there any roles that your group spent too much time attending to?**

## Communicating About Mathematics

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Marcella's Bagels

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## 1-2-3-4 Puzzle

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Uncertain Answers

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Extended Bagels

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## The Chefs' Hot and Cold Cubes

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Do It the Chefs' Way

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## POW 2: Checkerboard Squares

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## You're the Chef

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Investigations

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Consecutive Sums

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Add It Up

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Group Reflection

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

That's Odd!

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Pattern Block Investigations

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## Degree Discovery

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Polygon Angles

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## An Angle Summary

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Putting It Together

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Squares and Scoops

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Another In-Outer

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Diagonally Speaking

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

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## The Garden Border

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## Border Varieties

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

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## Patterns Portfolio

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Overview

Overview for The Overland Trail unit of IMP 2nd edition.

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Pacing Guides

Pacing guides for The Overland Trail unit of IMP Year 1 (2nd edition)

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## The Overland Trail Calculator Guide

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## A Journey Back in Time

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Crossing the Frontier

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Just Like Today

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## Overland Trail Families

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## Creating Families

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Overland Trail Names

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## POW 6: The Haybaler Problem

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[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Hats for the Families

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[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Family Constraints

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Planning for the Long Journey

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Overland Trail Price List

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## The Search for Dry Trails

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Setting Out with Variables

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Shoelaces

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Laced Travelers

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## To Kearny by Equation

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## The Vermillion Crossing

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Ox Expressions

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Ox Expressions at Home

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## If I Could See This Thing

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## The Graph Tells a Story

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Wagon Train Sketches and Situations

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Graph Sketches

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## In Need of Numbers

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[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## The Issues Involved

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Out Numbered

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## From Rules to Graphs

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## POW 7: Around the Horn

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## You're the Storyteller: From Rules to Situations

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Traveling at a Constant Rate

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Previous Travelers

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Broken Promises

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Sublette's Cutoff

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Who Will Make It?

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## The Basic Student Budget

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## Following Families on the Trail

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Graphing Calculator In-Outs

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Fort Hall Businesses

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Sublette's Cutoff Revisited

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## The Basic Student Budget Revisited

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## POW 8: On Your Own

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## All Four, One

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Travel on the Trail

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## Moving Along

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## All Four, One — Linear Functions

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Straight-Line Reflections

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Reaching the Unknown

### Intent

The activities in **Reaching the Unknown** support students' understanding of linear functions and their representations by emphasizing symbolic representations and connecting this work to that done in earlier activities.

### Mathematics

The emphasis of these activities is on symbol sense. Students focus on symbolic representations of linear functions and on developing a meaningful understanding of some of the symbol-manipulation procedures used to solve equations. The context of the unit, the distributive property, and a mathematical model for symbol manipulation all help support these procedures. The context supports understanding of what the symbols and their relationships represent. The distributive property allows students to derive equivalent equations. The “mystery bags” model, a pan-balance model of equations, gives students a metaphor for symbol manipulation.

Flexible algebraic thinking includes the processes of “doing” and “undoing,” such as the ability to find equations with given solutions as well as to find solutions to given equations. In these activities, students will work explicitly on developing this algebraic flexibility.

### Progression

Students will answer a variety of questions by writing and solving symbolic equations and systems of equations. They will revisit previous questions solved graphically and solve them symbolically. In addition, they will present their results on the final POW of the unit, compile unit portfolios, and complete in-class and take-home unit assessments.

[Fair Share on Chores](#)

[Fair Share for Hired Hands](#)

[More Fair Share on Chores](#)

[More Fair Share for Hired Hands](#)

[Water Conservation](#)

[The Big Buy](#)

[The California Experience](#)

[Getting the Gold](#)

[The Mystery Bags Game](#)

[More Mystery Bags](#)

[Scrambling Equations](#)

[More Scrambled Equations and Mystery Bags](#)

[Family Comparisons by Algebra](#)

[Starting Over in California](#)

[Beginning Portfolios](#)

[The Overland Trail Portfolio](#)

## Fair Share on Chores

### Intent

In this activity, students write equations, express them in equivalent form, and graph them. Two important concepts are reinforced in the process: abstracting a problem to an algebraic equation and reading numeric information from a graph. Students convert a linear equation into  $Y =$  form, conducive to use on the graphing calculator. The activity sets the stage for students to make meaning of graphical solutions to systems of equations.

### Mathematics

Students examine possible solutions to a linear equation that involves two variables and explore how to express that equation by giving one variable in terms of the other. They also solve equations for one variable in terms of the other, based on reasoning in the problem context. Finally, they graph linear conditions on the calculator and use this tool to find solutions to an equation.

### Progression

Students will work on the activity in groups and then share their results with the class. This activity sets the stage for lending graphical meaning to solving systems of equations. Students will revisit this situation in **More Fair Share on Chores**.

### Approximate Time

30 minutes

### Classroom Organization

Groups, followed by whole-class discussion

## Doing the Activity

Read the introduction as a class. If you have a map, you might point to the split between the California and Oregon Trails.

You may want to have a volunteer offer a sample solution for Question 1 as a way to get the class started. You may need to prompt students to consider values other than whole numbers.

Circulate as groups work. In Question 2, students are asked to express a problem condition by using an equation. Although this may seem straightforward, translating words into symbols can be a complex process. If an entire group is stumped, remind them of the strategy of identifying and writing in words the arithmetic used. In this case, it is likely that students are performing an arithmetic operation like “multiply the girls’ hours by 2 and the boys’ hours by 3, add the results, and see if the total equals 10.”

If groups are stuck on Question 3 or 4, focus them on the arithmetic. **If each girl’s shift was four hours, how would you find the length of each boy’s shift?** Help them to translate their verbal reply into symbolic form—an equation like  $B = \frac{10-2G}{3}$ .

## Discussing and Debriefing the Activity

Begin the discussion by asking students to compile their possible answers to Question 1 into an In-Out table. Because students will be expressing the boys’ shift length, **B**, in terms of the girls’ shift length, **G**, the girls’ shift length should go in the **In** column. Have students plot these points.

If students had trouble with Question 2, ask them to use a number pair from the table and express in words how it gives the desired total of ten hours. They will probably reply by saying something to this effect: “Multiply the length of a girl’s shift (such as  $\frac{1}{2}$ ) by 2 and the length of a boy’s shift (such as 3) by 3 and add.” Essentially, they have given you the equation; they just need to restate it symbolically.

Help students to see that the purpose of Questions 3 and 4 is to express the equation in the form  $B =$  so they can graph it on the calculator. Ask, **Why might you want to express one variable in terms of the other? If you know the length of a girl's shift, how would you use it to find the length of a boy's shift?** Otherwise, they could stick with the equation  $2G + 3B = 10$ , which is a more natural way to think about the problem.

Here are some observations that might arise in the discussion.

- The longer the shift for each girl, the shorter the shift for each boy.
- There is a maximum possible shift length for each group.
- The points that fit the equation lie on a straight line.
- Any point that is on the graph fits the equation, and vice versa.

Post the equation from Question 4 so students can refer to it in the next activity.

## Key Questions

**If each girl's shift was four hours, how would you find the length of each boy's shift?**

**How can you check whether a number pair fits the condition?**

**Why might you want to express one variable in terms of the other?**



## Fair Share for Hired Hands

### **Intent**

Students examine possible solutions to a linear equation in two variables and explore how to express that equation as one variable in terms of the other. The activity focuses on the connection between an equation and its graph to further set the stage for considering the graphical meaning of the solution to a system of equations.

### **Mathematics**

Students continue work with all four representations of linear functions. Most importantly, they write a rule in  $Y=$  form and consider the meaning of specific points on the line.

### **Progression**

Students use a context to derive a set of ordered pair solutions, plot these pairs, and determine an equation for the linear relationship. Again, students are asked to convert the equation into  $Y=$  form. Finally, a class discussion emphasizes the meaning of the points along the graph of the line. This sets the stage for the next activities, in which students consider the meaning of a point of intersection.

### **Approximate Time**

5 minutes for introduction

20 minutes for activity (at home or in class)

15 minutes for discussion

### **Classroom Organization**

Individuals, followed by whole-class discussion

## Doing the Activity

Let students know that this activity is similar to the previous activity, **Fair Share on Chores**. Although students have graphing calculators available, suggest they initially draw the graph on paper.

## Discussing and Debriefing the Activity

Have students compare ideas and answers in their groups. You might have several students put their solutions for Question 1 and 2 on the board. After a few minutes, gather the class and have some discussion of the responses.

Next, turn to Question 4, eliciting verbal descriptions of how to get **Y** from **X**, and comparing the different versions.

Discuss the equations students wrote for Question 5, which probably look much like  $Y = \frac{20-3X}{4}$ . As a class, test some of the points students found from their graphs in Question 3.

At this point you can ask the class, **What is the relationship between fitting the equation and being on the graph?** Students should come to this conclusion:

**Every point on the graph represents a solution to the equation, and every solution to the equation corresponds to a point on the graph.**

Essentially, this is the definition of a graph.

Then ask, **Is every solution to the problem on the graph? Does every point on the graph represent a solution to the problem?**

The discussion should bring out the fact that a point on the graph, which represents a solution to the equation, does not necessarily represent a solution to the problem. For example, the point represented by **X** = 4 and **Y** = 2 is on the graph and fits the equation, but is not a solution to the problem, because the experienced workers should have a higher pay rate than the inexperienced workers.

This tells us that even though the graph represents the equation, the equation itself does not precisely represent the problem. The graph for the actual problem is only a portion of the line. In other problems, only points with whole-number coordinates will fit the problem.

## **Key Questions**

**How did you get  $Y$  from  $X$  ?**

**What is the relationship between fitting the equation and being on the graph?**

**Is every solution to the problem on the graph?**

**Does every point on the graph represent a solution to the problem?**

## More Fair Share on Chores

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## More Fair Share for Hired Hands

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## Water Conservation

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## The Big Buy

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## The California Experience

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## Getting the Gold

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[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## The Mystery Bags Game

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## More Mystery Bags

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## Scrambling Equations

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## More Scrambled Equations and Mystery Bags

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## Family Comparisons by Algebra

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## Starting Over in California

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## Beginning Portfolios

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

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## The Overland Trail Portfolio

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## Overview

Overview for the Shadows unit of IMP 2nd edition.

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Pacing Guides

### Pacing guides for the Shadows unit of IMP Year 1 (2nd edition)

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

What is a Shadow?

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## How Long is a Shadow?

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## Experimenting with Shadows

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## The Shadow Model

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## POW 13: Cutting the Pie

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## Poetical Science

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

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## Shadow Data Gathering

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## An N-by-N Window

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## Working with Shadow Data

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## More About Windows

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## The Shape of It

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## Draw the Same Shape

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## How to Shrink It?

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## The Statue of Liberty's Nose

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

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## Make It Similar

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## POW 14: Pool Pockets

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## A Few Special Bounces

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## Ins and Outs of Proportion

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## Similar Problems

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## Inventing Rules

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[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Polygon Equations

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## Triangles Galore

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Triangles Versus Other Polygons

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Angles and Counterexamples

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Why Are Triangles Special?

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## More Similar Triangles

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Are Angles Enough?

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## In Proportion

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## What's Possible?

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## Very Special Triangles

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Angle Observations

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## More About Angles

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## POW 15: Trying Triangles

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

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## Inside Similarity

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## A Parallel Proof

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Angles, Angles, Angles

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## The Lamp Shadow

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## Bouncing Light

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Now You See It, Now You Don't

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Mirror Magic

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Mirror Madness

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## A Shadow of a Doubt

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## To Measure a Tree

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## POW 16: Spiralaterals

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## More Triangles for Shadows

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## The Sun Shadow

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## The Sun Shadow Problem

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Right Triangle Ratios

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Sin, Cos, and Tan Revealed

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Homemade Trig Tables

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Your Opposite is My Adjacent

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## The Tree and the Pendulum

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Sparky and the Dude

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.



## A Bright Sunny Day

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link

[<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## Beginning Portfolio Selection

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.

## "Shadows" Portfolio

The Interactive Mathematics Program teacher materials have been moved to the Key Curriculum Press website. This link [<http://www.keypress.com/keyonline>] will take you to the login page. After creating an account for yourself, you'll be able to access the full IMP Teacher's Guide.